A Disagreement with Dirac's Law

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A fundamental basis of the theory of quantum mechanics is Dirac's law. This is based on the observation for low z values that the velocity of the electron in a 1s orbit is proportionate to z. That is, the z=1 Hydrogen electron has a velocity of c/137.036 while the z=2 Helium electron has a velocity of 2c/137.036 (in each case relative to the nucleus). This was used to extrapolate velocities for all other z values, from which the energy of the electron could be calculated. Bohr approximated this non-relativistically, and Dirac adjusted Bohr's work to account for relativity. Unfortunately, this does not match what the astronomers were doing.

The electron's motion produces a centripetal effect pushing the electron outward counterbalancing the electro-dynamic pull inward. Classically, $F_c=mv^2/r$. Analyzing Dirac's work, it can be seen he treated this as $F_c=m_0v^2/r$. That is, his calculations imply the centripetal force is based on the rest mass. Worse, the form for the n shell electron is often given as C (circumference) = nλ (shell number * wavelength), when it should be λ=nC. That is, the wavelength (which is hc/E_K) is n times the circumference, not the circumference over n. For instance the 2s wavelength in Hydrogen is 4 times the 1s length, since it has a quarter the kinetic energy, while the radius of its orbit is twice the radius of the 1s electrons orbit.

From the behavior of small bodies near large bodies in the heavens (the asteroid Icarus when near the sun, for instance), it can be seen that the centripetal effect is proportionate to the effective mass. That is $F_c=m_vv^2/r$. This can be simplified, since the middle term is twice the kinetic energy, to $F_c=2E_k/r$.

Applying this form to atoms, it is possible to calculate velocities from energy, rather than vice-versa. From Dirac, at z=100 (Fermium) v=100c/137.036 which is about 0.7297c. Calculated from energy, E_k=10000*13.6057eV or 136.057 KeV with a rest energy of the electron of 510.999 KeV. Substituting through the Lorentz equation, we get a velocity of 0.6135c. This form allows for any z without exceeding the speed of light.

At first blush, this flies in the face of observation. Fusion reactions have shown that large z values are not permitted. The actual problem does not lie in the speed of the electron, but rather in its size.

An interpretation of Compton's scattering produces an electron with an effective radius of 386 fm. Putting that in perspective, the Hydrogen atom has a radius of 52918 fm, and no known nucleus has a radius over 12 fm (with a radius under 8 fm being typical). Solving the energy based force balance equation produces an orbital radius $r=na_0/z$ where a_0 is Bohr's radius. For n=1 (the 1s case), the radius of orbit is Bohr's radius over z (the charge on the nucleus).

At z=100, r is 529.18 fm. That means the surface of the electron clears the surface of the nucleus by at least 130 fm. But at z=125, r is 423 fm (with 25 – 29 fm clearance). Finally, at z=137.036, the surface of the electron is passing through the center of the nucleus, which is absurd. Somewhere around z=132 (untribium) the electron's surface is just skimming the surface of the nucleus. At higher z values, the electron crashes into the nucleus in normal 1s orbit. This could be compared to solving the force equations to show Icarus could orbit within 100,000 km of the center of the sun, without taking into account the size of the sun (which is nearly 1,400,000 km in diameter).

This form of the electron's orbit gives $E_k=hcz\alpha/4\pi rn$ (where α is the fine structure constant). Typically, $2\pi rn$ (wavelength) is combined into the single symbol λ. It should be recalled that r is the distance between the 2 charges, not the observed radius of the electrons orbit, which is the slightly smaller distance between the center of mass and the electron. The kinetic energy is likewise the systemic kinetic energy, most of which (13.5983 eV) appears on the electron (since the proton is trudging along in its orbit ~28 femtometers in radius at $\sim c/2.5*10^5$, for a kinetic energy of .0074 eV). In cases where the 2 charges have more similar masses, the energy is distributed more equitably (Muonic Hydrogen would have 12.23 eV on the Muon, and 1.38 eV on the Proton). For heavier nuclei almost all the energy is in the electron. Since for the spherical case, $r=na_0/z$ we have $E_k=hcz^2\alpha/4\pi a_0n^2$. This is usually simplified to 13.6057 $eV*z^2/n^2$.

Let's look at the simplest comparable case: a Tritium atom (Hydrogen 3) decaying. The nucleus emits an electron (at high enough energy to leave the vicinity) and becomes a Helium-3 ion. Before the decay, the orbiting electron had a kinetic energy of 13.603 eV (and a rest energy of 510998.918 eV). Substituting through the Lorentz equation, this is a velocity of c/137.052 (relative to the center of mass). After the decay, the electron accelerates inward to a new lower orbit, with half the radius and around twice the velocity, giving a new kinetic energy of 54.413 eV. Again substituting through Lorentz, we get a new velocity of c/68.529 or 2c/137.059. Almost all the acceleration went into a change in velocity, but some (40.8 eV/c^2 worth) added to the effective mass.

Thus far z has been treated as a simple integer. A better interpretation is that z* (the effective z) is the central z less the shielding effect of the other electrons. For an electron in shell n, z* is z less the sum of the electrons in the lower shells, and less some partial amount for the electrons in its own shell. A classic approximation yields z* of about 1.70 for the Helium 1s electrons (with 79.0 eV of kinetic energy shared between them). Applied to our crashing electrons, the effect is marginal. Instead of the electron's surface passing through the center of the nucleus at z=137.036 it passes through it around 137.5. Not a big improvement.

Z* can also be seen from similar cases in chemistry, such as 2 protons near an oxide ion. It is easy to strip off a single proton forming a hydroxide ion, but it is very difficult to strip off the second proton. Z* is around 1.12 for the hydroxide ion (hence around 1.9 for the oxide ion), and the whole H_2O complex can easily add a 3rd proton, where Helium can only attract 2 electrons.

Several other small effects do need to be accounted for near the extreme case. The most obvious is tidal forces. These are proportionate to the cube of the difference in distance to the center of the orbit from the point nearest to the center and the point furthest from the center. In the case of the electron, its inner surface is 143 fm from the center of the nucleus in the z=100 case, while its outer surface is 917 fm away. The ratio of distances increases exponentially for high z values. It may be necessary to calculate a tidally adjusted centripetal effect for the electron to account for the bulges that could potentially result. At a minimum, the outer surface is moving faster than the inner surface, so has a higher effective "mass" than a calculation based solely on center of mass calculations suggest. A Roche's limit may even be possible (where the electron flies to pieces from the tidal effects alone).

Another small effect is the variation in charge distribution across the face of the nucleus. Points where the charge is more concentrated would lead to higher velocities in the orbiting electron, while dilute regions would produce a slower velocity. This effect averages out, except the electron may bump into the nucleus as a result. Other calculations have suggested an electro-centripetal effect, but even at z near 137 this is less than a half percent of the magnitude of the mass-centripetal effect just calculated (so can probably be ignored for now). Entanglement effects have also been theorized, but again only at extreme energies which an orbiting electron can't produce.

This argument treats the electron as a structure, with definite boundaries – which differs from the current view of quantum mechanics which treats the electron as a wave dispersed throughout its orbit. Such a structure has constituents in orbits, with calculable properties. Each view has virtues.

In conclusion: the theoretic underpinning of Quantum Physics includes Dirac's Law, which must now be re-examined. At low velocity (under c/7) it is correct to at least 2 digits, but at very high velocities it falls to pieces. The correct value of the energy for a 1s electron in high z elements is commonly known, and is within the shielded range of $z \geq z^* \geq z-.5$, which matches what is being suggested. The orbits of high velocity astronomical objects have been carefully measured and show a centripetal force proportionate to effective mass, not to rest mass.