An Alternative view of Nuclear Energy

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While it has been a long-standing convention to calculate the energy of isotopes based on how much energy would be released by merging n neutrons and z protons to form the atom, this does not jibe with the actual initial conditions. The early universe had a large quantity of hydrogen gas, which is basically a proton and an electron, and trivial amounts of other nuclei which contained neutrons.

A better alternative is to determine the energy content of the nuclei as a group of baryons and find the energy per baryon. An immediate problem arises due to lack of accurate data about nuclear mass. A large quantity of data has been generated about atomic mass – and almost all the isotopes have such data – but that includes the masking values of electrons orbiting the nucleus.

A method of getting around this is to estimate the energy content of the electrons and subtract that from the total energy content of the atom. For the small isotopes that can be done. However the total energy content of the electrons in large isotopes is not readily available. Since the total energy content of the electrons is primarily the rest mass of the electrons, a reasonable approximation can be done by assuming some value for the kinetic energy of the electrons.

The simplest method is to use a reasonable model of electron energy and test against known values. In this case the Bohr model is sufficient. Kinetic energy of the inner electrons of the heaviest known elements is less than half the mass equivalent. Using a simple formula, a reasonable approximation can be generated: the energy of electrons in shell n each have 13.606 eV $*(z-(\text{electrons in shells} <n)-(\text{half the electrons in shell n}))^2$.

The results from this method differ at the margin from results generated using the conventional method. In this case Iron-56 is the lowest energy nuclei, easily beating Nickel 62. In general, those with a low neutron fraction improved their ranking, while those with a high neutron fraction dropped. The lowest energy nuclei (those with less than 930.25 MeV per baryon) are shown below.

(Note – first set of columns is alphabetic by element name, second set is numeric by baryon count, third set is by energy per baryon. Each relevant isotope occurs in all 3 sets.)

Chromium	50	930.24312
Chromium	51	930.24433
Chromium	52	930.19205
Chromium	53	930.21911
Chromium	54	930.21220
Cobalt	56	930.24643
Cobalt	57	930.21034
Cobalt	58	930.22382
Cobalt	59	930.20497
Cobalt	60	930.23611
Cobalt	61	930.23625
Copper	61	930.23441
Copper	62	930.24160
Copper	63	930.21732
Copper	64	930.23970
Copper	65	930.23069
Iron	54	930.20579
Iron	55	930.20690
Iron	56	930.17407
Iron	57	930.20468
Iron	58	930.19289
Iron	59	930.24020
Iron	60	930.24863
Manganese	53	930.22069
Manganese	54	930.22820
Manganese	55	930.21203
Manganese	56	930.24923
Nickel	58	930.20839
Nickel	59	930.21445
Nickel	60	930.18050
Nickel	61	930.20615
Nickel	62	930.18620
Nickel	63	930.22654
Nickel	64	930.22155
Titanium	48	930.24929
Titanium	50	930.24030
Vanadium	51	930.23963
Zinc	64	930.22262
Zinc	65	930.24359
Zinc	66	930.21727
Zinc	68	930.23853

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Chromium	50	930.24312
Vanadium	51	930.23963
Chromium	51	930.24433
Chromium	52	930.19205
Chromium	53	930.21911
Manganese	53	930.22069
Iron	54	930.20579
Chromium	54	930.21220
Manganese	54	930.22820
Iron	55	930.20690
Manganese	55	930.21203
Iron	56	930.17407
Cobalt	56	930.24643
Manganese	56	930.24923
Iron	57	930.20468
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Iron	58	930.19289
Nickel	58	930.20839
Cobalt	58	930.22382
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Nickel	59	930.21445
Iron	59	930.24020
Nickel	60	930.18050
Cobalt	60	930.23611
Iron	60	930.24863
Nickel	61	930.20615
Copper	61	930.23441
Cobalt	61	930.23625
Nickel	62	930.18620
Copper	62	930.24160
Copper	63	930.21732
Nickel	63	930.22654
Nickel	64	930.22155
Zinc	64	930.22262
Copper	64	930.23970
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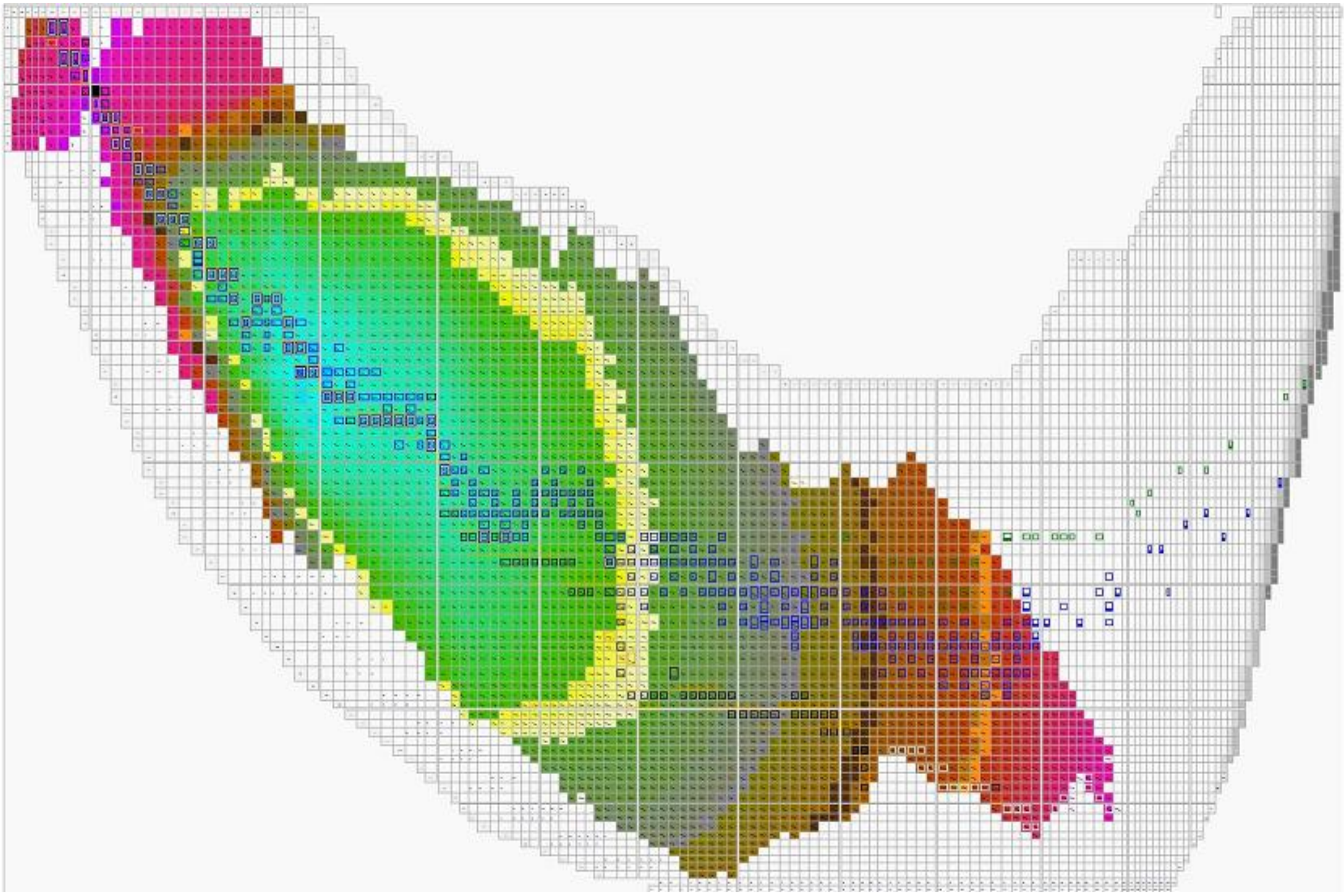
Iron	55	930.20690
Nickel	58	930.20839
Cobalt	57	930.21034
Manganese	55	930.21203
Chromium	54	930.21220
Nickel	59	930.21445
Zinc	66	930.21727
Copper	63	930.21732
Chromium	53	930.21911
Manganese	53	930.22069
Nickel	64	930.22155
Zinc	64	930.22262
Cobalt	58	930.22382
Nickel	63	930.22654
Manganese	54	930.22820
Copper	65	930.23069
Copper	61	930.23441
Cobalt	60	930.23611
Cobalt	61	930.23625
Zinc	68	930.23853
Vanadium	51	930.23963
Copper	64	930.23970
Iron	59	930.24020
Titanium	50	930.24030
Copper	62	930.24160
Chromium	50	930.24312
Zinc	65	930.24359
Chromium	51	930.24433
Cobalt	56	930.24643
Iron	60	930.24863
Manganese	56	930.24923
Titanium	48	930.24929

This energy mapping does have some drawbacks. In many cases capturing an electron increases the stability of the resulting nucleus while also increasing the energy per baryon, for example Iron 55 versus Manganese 55 (which is still less than a third the mismatch of the standard form). On the plus side, it provides insight into the relative long life of the heavier isotopes. Uranium 238 only has 931.493 MeV per baryon, far less than the 931.845 of Helium 4. A complete table is available online giving energy for the 3200 known isotopes. This is in n-2z (y-axis) by a/3, a useful form for such data. Lowest energy for a baryon count is outlined in blue. The 2 lowest ratios for an element are outlined in black (for even N) and then orange (odd N).



Here is a comparison between this form of the data and the binding energy per baryon classically. Sum approximates the weighted average of protons and neutrons the classic would use if the electrons effect were taken into account. It tracks closely to the ratio of neutrons to protons, with rising z a secondary effect.

Element	**Weight**	**Energy/ Baryon**	**Classic Energy**	**Sum**	**N/Z**
Helium	4	931.84473	7.073915	938.91864	1.000
Nickel	58	930.20839	8.732038	938.94043	1.071
Cobalt	56	930.24643	8.694824	938.94125	1.074

Iron	54	930.20579	8.736343	938.94213	1.077
Chromium	50	930.24312	8.700981	938.94410	1.083
Copper	61	930.23441	8.715505	938.94992	1.103
Nickel	59	930.21445	8.736568	938.95102	1.107
Cobalt	57	930.21034	8.741856	938.95220	1.111
Iron	55	930.20690	8.746559	938.95346	1.115
Manganese	53	930.22069	8.734132	938.95482	1.120
Chromium	51	930.24433	8.711955	938.95628	1.125
Zinc	64	930.22262	8.735897	938.95852	1.133
Copper	62	930.24160	8.718074	938.95967	1.138
Nickel	60	930.18050	8.780754	938.96125	1.143
Cobalt	58	930.22382	8.738944	938.96276	1.148
Iron	56	930.17407	8.790322	938.96439	1.154
Manganese	54	930.22820	8.737922	938.96612	1.160
Chromium	52	930.19205	8.775944	938.96799	1.167
Zinc	65	930.24359	8.724257	938.96785	1.167
Copper	63	930.21732	8.752131	938.96945	1.172
Nickel	61	930.20615	8.765006	938.96998	1.179
Titanium	48	930.24929	8.722905	938.97220	1.182
Cobalt	59	930.20497	8.768010	938.97298	1.185
Iron	57	930.20468	8.770248	938.97493	1.192
Manganese	55	930.21203	8.764988	938.97702	1.200
Zinc	66	930.21727	8.759631	938.97690	1.200
Copper	64	930.23970	8.739067	938.97877	1.207
Chromium	53	930.21911	8.760155	938.97926	1.208
Nickel	62	930.18620	8.794546	938.98075	1.214
Vanadium	51	930.23963	8.742051	938.98168	1.217
Cobalt	60	930.23611	8.746742	938.98285	1.222
Iron	58	930.19289	8.792220	938.98511	1.231
Manganese	56	930.24923	8.738299	938.98753	1.240
Copper	65	930.23069	8.757094	938.98778	1.241
Chromium	54	930.21220	8.777913	938.99011	1.250
Nickel	63	930.22654	8.763486	938.99003	1.250
Cobalt	61	930.23625	8.756149	938.99240	1.259
Zinc	68	930.23853	8.755677	938.99421	1.267
Iron	59	930.24020	8.754742	938.99494	1.269
Titanium	50	930.24030	8.755621	938.99592	1.273
Nickel	64	930.22155	8.777461	938.99901	1.286
Iron	60	930.24863	8.755836	939.00447	1.308
Uranium	238	931.49291	7.570120	939.06303	1.587